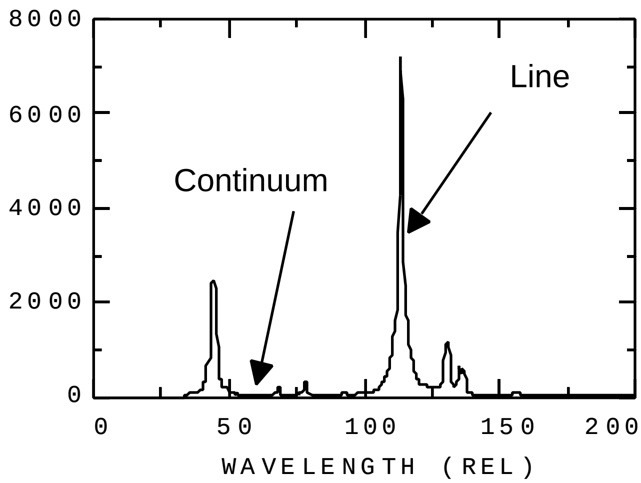
Electromagnetic radiation from electron ion collisions (bremsstrahlung, recombination)

Contents

Introduction

The goal of spectroscopy as a plasma diagnostic is to interpret the emission from a plasma in terms of the plasma properties. The range of photon energies of interest is limited to 1 eV and 12 keV as this range is particularly sensitive to the properties of the plasma. This is an example of a fragment of the spectrum



The spectrum is composed of lines and continuum. The plasma that emitted this spectrum was composed of fully stripped ions, free electrons, atoms, and ions with bound electrons. Atoms and partially stripped ions immersed in the plasma emit discrete spectra (lines) and continuous spectra. The structure of both is affected by interactions with the plasma particles and hence depends to some degree on the thermodynamic properties of those particles. Unbound electrons emit continuous spectra which is also affected by interactions with plasma particles.

Topical areas

Collection of light

Transmission/absorption of materials
Discrete optics
Fiber optics
Problems with spatial asymmetries and techniques for overcoming them based on multiple sight lines and on active spectroscopy

Selection of a portion of the spectrum

Diffraction gratings
Interference filters (include multilayers)
Absorption filters

Analysis

Requirements of a good spectroscopic diagnostic.

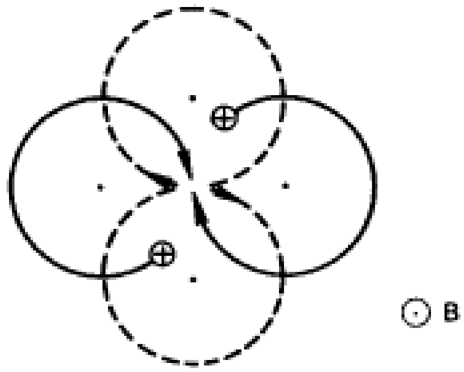
a clearly defined plasma property that is to be measured
a mechanism for impressing the property on the spectrum
a means for collecting light from the interesting region of the plasma
a means for isolating the spectral component that contains the information
an analysis that allows extraction of the information pertaining to the plasma property, and the relation between this information and the plasma property

Theory

bremsstrahlung

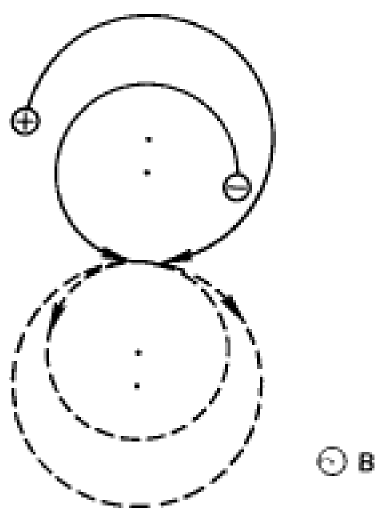
A free electron accelerates in the E-field of a charged particle. The result is free-free or free - bound (electron is captured by the ion). The latter (free-bound) is called recombination, and the total energy less than zero. Electron-electron collisions can be ignored unless they are both relativistic. Binary collisions between identical particles are such that there is no net acceleration of the center of mass or center of charge, so to lowest order the fields from the two particles cancel.

Need quantum treatment. But standard is classical. Plasma is composed of ions and electrons. All collisions are between charged particles. There is a big difference between like particle collisions (e.g. ion-ion or electron-electron) and unlike (electron-ion or ion-electron). Consider two identical particles colliding. If it is a head-on collision, then particles emerge with velocities reversed, they interchange orbits. Therefore the two guiding centers remain unchanged. The result is the same for glancing orbits, in which the trajectories are almost unchanged. The worst case is a 90 degree collision, in which velocities are changed 90 0 in direction. The orbits are then the dashed orbits, but the center of mass is unchanged. Thus collisions between like particles cause very little diffusion.

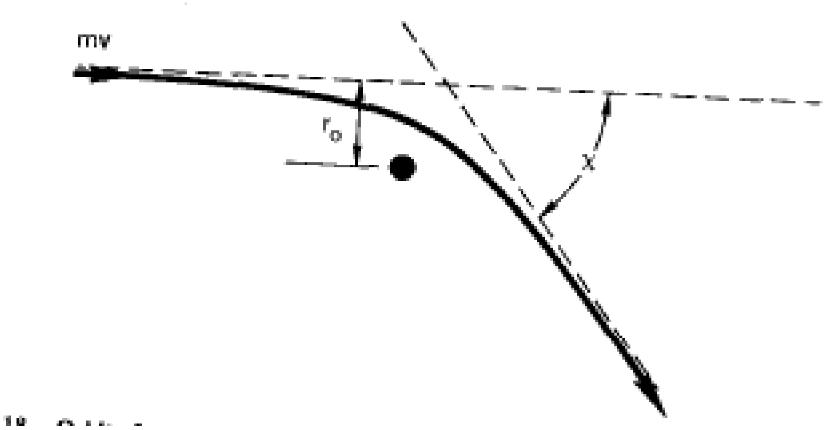


Note that there is a spreading. Considering the reverse collision (broken to solid line) then there is a flux across some line drawn in the x direction (left to right). The resolution of this paradox is that the net flux involves an averaging over all collisions, including the pair of collisions (forward and backward), and that the frequency of collisions is proportional to th product of the densities at the locations of the two guiding centers.

When two unlike particles collide, the situation is different. Consider the worst case 180 degree collision, in which the particles emerge with their velocities reversed. The figure shows the situation: They must continue to gyrate around the line of force in the correct direction - the only solution is that both guiding centers move in the same direction



Now consider an electron colliding with an ion. The electron is gradually defected by the Long Range Coulomb force. An electron of velocity v approaches a fixed ion of charge e. In the absence of the charge the closest approach is r_0 In the presence of the charge, the electron is deflected by an angle c. This is related to r_0.



The force is felt over a time T = r_0/v (approximately)

$$F = -\frac{e^2}{4\pi\varepsilon_0 r^2}$$

The momentum change is approximately

$$\Delta(mv) = |FT| = \frac{e^2}{4\pi\varepsilon_0 r_0 v}$$

For a large angle collision (say 90) the change in momentum *mV* is about equal to *mV*. Then

$$\Delta(mv) \approx mv \approx \frac{e^2}{4\pi\varepsilon_0 r_0 v}$$

$$r_0 \approx \frac{e^2}{4\pi\varepsilon_0 mv^2}$$

The collision cross section is

$$\sigma = \pi r_0^2 \approx \frac{e^4}{16\pi\varepsilon_0^2 m^2 v^4}$$

The collision frequency is

$$\nu_{ei} = n\sigma v = \frac{ne^4}{16\pi^2\varepsilon_0^2 m^2 v^3}$$

Consider simple situation in which a flux $\Gamma = nv$ of particles of mass m, density n, velocity v, is incident on a half space of stationary, infinite mass particles of density n_g. The relative velocity $v_r = v$. Let dn be the number of incident particles per unit volume at x that undergo an interaction within a distance dx, removing them from the incident beam. Then

$$dn = -\sigma n n_g dx$$

minus sign denotes removal. Removal might be scattering by $\pi/2$ or more, ionization, etc. Then multiply by v to get

$$d\Gamma = -\sigma\Gamma n_g dx$$

A simple interpretation of σ is for elastic spheres. Incident radius a_1, target radius a_2. In a distance dx their are ndx targets within a unit area perpendicular to the incident velocity (i.e. along x). Draw a radius $a_{12} = a_1 + a_2$ in the x = constant plane about each target. A collision occurs if the centers of incident and target particles are within this radius. Hence the fraction of the unit area for which a collision occurs is $n_g dx \pi a_{12}^2$. The fraction of incident particles which collide within dx is

$$\frac{d\Gamma}{\Gamma} = \frac{dn}{n} = -\sigma n_g dx$$

with $\sigma = \pi a_{12}^2$. Integrate to get flux:

$$\Gamma = \Gamma_0 e^{-x/\lambda}$$
$$\lambda = \frac{1}{n_g \sigma}$$

λ is the mean free path. The time between interactions is $\tau = \lambda/v$. The inverse is the collision frequency

$$\nu = \frac{1}{\tau} = n_g \sigma v$$)

The plasma resistivity is defined in terms of a fluid by writing the change of momentum as

$$P_{ei} = mn(v_i - v_e)\nu_{ei}$$

Expecting P_{ei} to be proportional to the density of electrons and scattering centers (ions) and the relative velocity and the Coulomb force (i.e. e^2) then

$$P_{ei} = \eta e^2 n^2 (v_i - v_e)$$

and the "resistivity" is

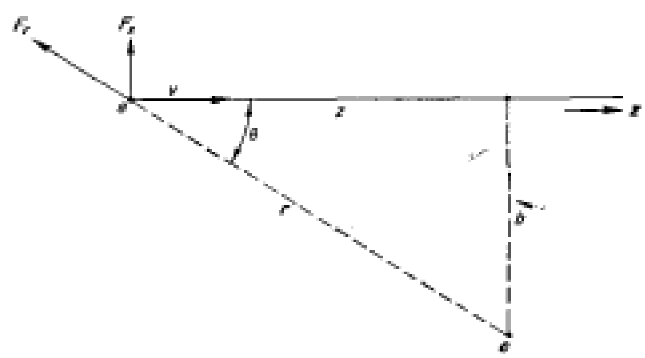
$$\eta = \frac{m\nu_{ei}}{ne^2}$$

Then

$$\eta = \frac{m\nu_{ei}}{ne^2} = \frac{e^2}{16\pi\varepsilon_0^2 m v^3}$$

For a Maxwellian, in which small angle collisions are more frequent than the assumed large angle collisions, the cumulative effect is larger, and

$$\eta = \frac{\pi e^2 \sqrt{m}}{(4\pi\varepsilon_0)^2 (kT_e)^{3/2}} \ln(\Lambda)$$



For multiple collisions, consider figure above. r is distance between particles and b their impact parameter. The Coulomb force is F_r and has a component F_x at right angles to the z direction, the direction in which the scattered particle is moving. Then momentum change if scattered particle in x direction is

$$\Delta p_x = \int_0^\infty F_x dt = \int_0^\infty F_r \sin(\theta) dt = \frac{e^2}{b^2} \int_0^\infty \sin^2(\theta) \sin(\theta) dt$$

since $b/r = \sin\theta$. Now $dz/dt = v$, so $dt = dz/v$, and since $b/z = -\tan\theta$, we have

$$dt = \frac{b\,\mathrm{cosec}^2(\theta)\,d\theta}{v}$$

and

$$\Delta p_x = \frac{e^2}{bv}\int_0^{\pi}\sin(\theta)\,d\theta = \frac{2e^2}{bv}$$

Now consider many collisions. Total momentum change along *x* axis is sum of individual momentum changes. If many scattered particles are considered, each having the same initial velocity and direction of motion and making the same number of collisions *N*, an average momentum change can be found.

$$\Delta p_x = (\Delta p_x)_1 + (\Delta p_x)_2 + .. + (\Delta p_x)_j + .. + (\Delta p_x)_N$$
$$(\Delta p_x)^2 = (\Delta p_x)_1^2 + (\Delta p_x)_2^2 + ..(\Delta p_x)_j^2 + .. + (\Delta p_x)_N^2$$
$$+(\Delta p_x)_1(\Delta p_x)_2 + ... + (\Delta p_x)_j(\Delta p_x)_k + ..$$

If $(\Delta p_x)_j$ in each collision is small, the average will be the sane for all collisions. Also, taking into account the random deflections, the cross products will vanish when averaged over all particles. The same is true for other directions(i.e. y), so that

$$\left\langle(\Delta p)^2\right\rangle = N\left\langle(\Delta p)_j^2\right\rangle$$
$$d\left\langle(\Delta p)^2\right\rangle = \left\langle(\Delta p)_j^2\right\rangle dN$$

If *n* is number density of scattering (field) particles then number d*N* contained in a cylindrical shell of length *l*, radius *b*, thickness d*b* is

$$dN = 2\pi n l b\,db$$

so that

$$d(\Delta p)^2 = \left(\frac{2e^2}{bv}\right)^2 2\pi n l b\,db = \frac{8\pi e^4}{v^2}\,nl\,\frac{db}{b}$$

Now integrate over all values of impact parameter to get

$$\left\langle(\Delta p)^2\right\rangle = \frac{8\pi e^4}{v^2}\,nl\ln\Lambda; \quad \ln\Lambda = \frac{b_{\max}}{b_{\min}}$$

For scattered mass less than or equal to scattering particle, assume that when $<\Delta p^2>$ has increased to the value p^2, then particle has scattered through a large angle (90^0). l is the distance over which the cumulative effect of many small angle scattering is the same as one 90^0 scattering. Then $l = 1/(n\sigma_d)$. When $p = mv$ so that $<\Delta p^2> = (mv)^2$, with v an average, we get

$$(mv)^2 \approx \frac{1}{\sigma_d}\frac{8\pi e^4}{v^2}\ln\Lambda$$

$$\sigma_d \approx \frac{8\pi e^4}{m^2 v^4}\ln\Lambda$$

Use b_{max} as the 90^0 scattering, b_{min} the Debye shielding distance to get:

$$\Lambda = \frac{\lambda_D}{b_0} = \left(\frac{\varepsilon_0 kT}{n_e e^2}\right)^{1/2}\left(\frac{12\pi\varepsilon_0 kT}{e^2}\right) \quad \frac{(kT)^{3/2}}{n_e^{1/2}}$$

The actual trajectory is given by (for an impact parameter b and an incident velocity v_1)

$$r = \frac{b^2}{b_{90}(1+\varepsilon\cos\theta)}$$

$$b_{90} = \frac{Ze^2}{4\pi\varepsilon_0 m v_1^2}$$

$$\varepsilon = \sqrt{1+\left(\frac{b}{b_{90}}\right)^2}$$

The radiation from such an encounter over all solid angles can be calculated, as we know the orbit, and thus the acceleration. The free electron path is a conic section. The radiation over all solid angles (non relativistic) becomes:

$$\frac{dW}{d\omega} = \frac{e^2}{6\pi^2\varepsilon_0 c^3}\left|\int_{-\infty}^{\infty}\dot{\mathbf{v}}e^{i\omega t}dt\right|^2$$

Substitute for v as determined from the orbit and the energy equation into the Fourier integrals:

$$\frac{1}{2}m\left(\dot{r}^2+\left(r\dot{\theta}\right)^2\right)-\frac{Ze^2}{4\pi\varepsilon_0 r} = \frac{1}{2}mv_1^2$$

To calculate the radiation from a single electron with a random set of ions of density n_i, multiply by $n_i v_1$ and integrate over the impact parameter to get the power spectrum

$$\frac{dP}{d\omega} = n_i \mathrm{v}_1 \int_0^\infty \frac{dW}{d\omega}(\omega, \mathrm{b}) 2\pi b\, db$$

Result given by Kramers (1923) as

$$\frac{dP}{d\omega} = \left.\frac{dP}{d\omega}\right|_c G(u_{90})$$

$u_{90} = i\omega b_{90} / \mathrm{v}_1$ is a dimensionless frequency, and the frequency independent part is

$$\left.\frac{dP}{d\omega}\right|_c = \frac{16\pi Z^2 e^6 n_i}{3\sqrt{3}(4\pi\varepsilon_0)^3 m^2 c^3 \mathrm{v}_1}$$

and G (the Gaunt factor) is a parameter of order 1. In terms of the fine structure constant $\alpha = e^2 / (4\pi\varepsilon_0 \hbar c)$ and the classical electron radius $r_e = e^2 / (4\pi e_o m c^2)$ we have

$$\left.\frac{dP}{\hbar d\omega}\right|_c = n_i \mathrm{v}_1 \frac{16_i}{3\sqrt{3}} \alpha \frac{c^2}{\mathrm{v}_1^2} \pi (Z r_e)^2$$

Radiation occurs where the acceleration is largest, when the electron is close to the ion. Let subscript 0 refer to closest approach, then conservation of angular momentum gives $v_1 b = v_0 r_0$, and at this time the acceleration is

$$\dot{\mathrm{v}}_0 = \frac{Ze^2}{4\pi\varepsilon_0 m r_0^2}$$

The time duration of close approach is

$$\tau = \frac{2r_0}{\mathrm{v}_0}$$

Evaluate Fourier transform of ∂v/∂t: its peak value will occur when the oscillating part of exp(iωt) is synchronized with the variation in ∂v/∂t. For this frequency (ω_0) we find that

$$\int \dot{v} e^{i\omega t} dt \approx \dot{v}_0 \tau \approx \frac{2Ze^2}{4\pi\varepsilon_0 m v_1 b}$$

$$\left.\frac{dW}{d\omega}\right|_{\omega_0} \approx \frac{Z^2 e^6}{(4\pi\varepsilon_0)^3} \frac{8}{3m^2c^2} \frac{1}{b^2 v_1^2}$$

this gives approximate peak in spectrum. Shape give by distribution of b's and 's.

Limits of collisions

1) $b >> b_{90}$: straight-line collision

2) $b << b_{90}$: parabolic collision (returns in direction it came from)

Quantum mechanical corrections. Note u/v catastrophe: the integral of $dP/d\omega$ increases to infinity at high frequencies. The deBroglie wave number for electrons is

$$k_h = \frac{mv}{\hbar}$$

and wave nature of electron ignorable only if $k_h b >> 1$. In high frequency regime ($b << b_{90}$) the requirement on photon size is more important than the deBroglie wavelength. Therefore consider restriction on $k_h b$ only in low frequency $b > b_{90}$ limit. Write classical limit as requiring $k_h b_{90} >> 1$. em radiation quantum effects negligible only when photon energy is much less than initial particle energy. Sommerfeld (1934) performed complete quantum analysis.

Remember to integrate over a distribution function. Radiation per unit volume is obtained by

$$\int \frac{dP}{d\omega} f d\mathrm{v}$$

Use Maxwell-averaged Gaunt factor $\bar{g}$, in which the factor $\exp(-\hbar\omega/T)$ is taken out from G. Obtain the spectral power emitted into 4π st per unit frequency as

$$4\pi j(\omega) = n_e n_i Z^2 \left(\frac{e^2}{4\pi\varepsilon_0}\right)^3 \frac{16\pi}{3\sqrt{3}m^2c^3} \left(\frac{2m}{\pi T}\right)^{1/2} e^{-\frac{\hbar\omega}{T}} \bar{g}$$

Recombination radiation

Conurbation from free-bound. Consider collision at velocity v_1 and impact parameter b << b_{90}, i.e. a parabolic collision. The spectral energy density has been derived. For photon energy $\hbar\omega < (1/2)mv^2$, final state of electron is free. But for $\hbar\omega > (1/2)mv^2$ final state is bound, and must take up discrete energies

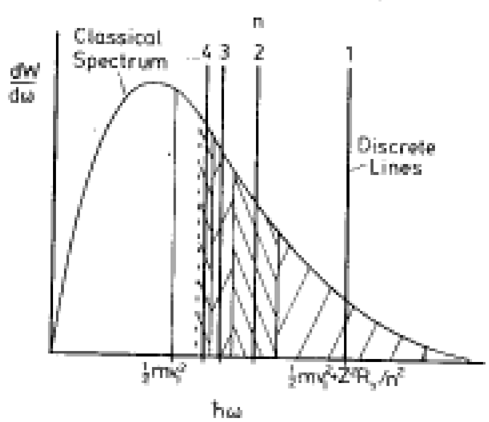
$$W_n = \frac{-R_y Z^2}{n^2} = -\frac{Z^2 e^4 m}{2(4\pi\varepsilon_0\hbar)^2}\frac{1}{n^2}$$

Therefore radiation is a set of discrete lines with

$$\hbar\omega = \frac{1}{2}mv^2 + \frac{R_y Z^2}{n^2}$$

(R_y = 13.6 eV). Energy in the nth line is the energy in the frequency range

$$\frac{1}{2}mv^2 + \frac{R_y Z^2}{\left(n+\frac{1}{2}\right)^2} \le \hbar\omega \le \frac{1}{2}mv^2 + \frac{R_y Z^2}{\left(n-\frac{1}{2}\right)^2}$$



Then the frequency width is

$$\Delta\omega_n \approx \frac{2R_y Z^2}{\hbar n^3}$$

and the energy in the nth line is given by the classical expression

$$\frac{dW(b,\mathrm{v})}{d\omega}\Delta\omega_n$$

Note the power spectrum is terminated at an energy $R_y/(1/2)^2$.

Now integrate over impact parameter to get total power in the nth line per electron of velocity *v* in a plasma of density n_i:

$$P_n = \left.\frac{dP}{d\omega}\right|_c \Delta\omega_n G_n = \frac{16\pi Z^2 e^6 n_i}{3\sqrt{3}(4\pi\varepsilon_0)^3 m^2 c^3 v_1}\frac{2Z^2 R_y}{\hbar n^3} G_n$$

To calculate the radiation from recombination for an electron distribution function f(ϖ) as a function of w for level ν, use $v = \left[2\left(\hbar\omega - Z^2 R_y / n^2\right)/m\right]^{1/2}$. For a Maxwellian plasma we have

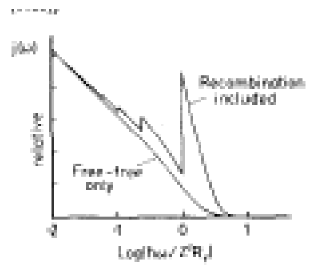
$$4\pi j(\omega) = n_e n_i Z^2 \left(\frac{e^2}{4\pi\varepsilon_0}\right)^3 \frac{16\pi}{3\sqrt{3} m^2 c^3}\left(\frac{2m}{\pi T}\right)^{1/2} e^{-\frac{\hbar\omega}{T}}\left[\frac{Z^2 R_y}{T}\frac{2}{n^3} G_n e^{\frac{Z^2 R_y}{n^2 T}}\right]$$

True if $\hbar\omega > Z^2 R_y / n^2$, otherwise = 0. No recombination occurs for a frequency < $Z^2 R_y / \left(\hbar n^2\right)$.

Low frequencies $\hbar\omega << Z^2 R_y$: only high n contribute; because of n^{-3} dependence the recombination effects are negligible. At high frequencies all n's contribute. Steps occur at the recombination edges, where $\hbar\omega = Z^2 R_y / n^2$, where the next level starts to contribute.

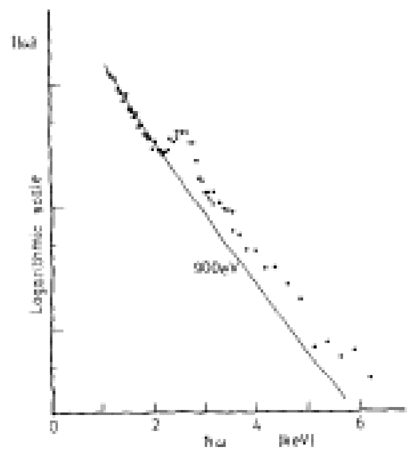
For incomplete ionized species, (high Z ions), assume previous treatment is O.K., except for lowest electron shell. Use ionic charge for *Z* rather than nuclear charge. i.e. assume perfect shielding of the nucleus by the bound electrons. Correct only if photon and electron energy are not much large than ionization potential. For lowest unfilled shell modify recomb. formulae because if this shell contains some electrons then there are less than the usual $2n^2$ holes available. χ_i is ionization potential, ξ is number of holes available (instead of $2n^2$)

$$j(\omega) = n_e n_i Z^2 \left(\frac{e^2}{4\pi\varepsilon_0}\right)^3 \frac{4}{3\sqrt{3} m^2 c^3}\left(\frac{2m}{\pi T}\right)^{1/2} e^{-\frac{\hbar\omega}{T}}$$
$$\times\left[g_{ff} + G_n \frac{\xi}{n^3}\frac{\chi_i}{T} e^{\frac{\chi_i}{T}} + \sum_{\nu = n+1}^{\infty} \frac{Z^2 R_y}{T}\frac{2}{\nu^3} G_\nu e^{\frac{Z^2 R_y}{\nu^2 T}}\right]$$



Temperature measurement.

Bremsstrahlung emitted over large frequency range. from plasma frequency (microwave) up to $\hbar\omega = T$ (x-ray). For $\hbar\omega > T$, both free-free and free-bound radiation intensities have a strong exponential dependence on the temperature.

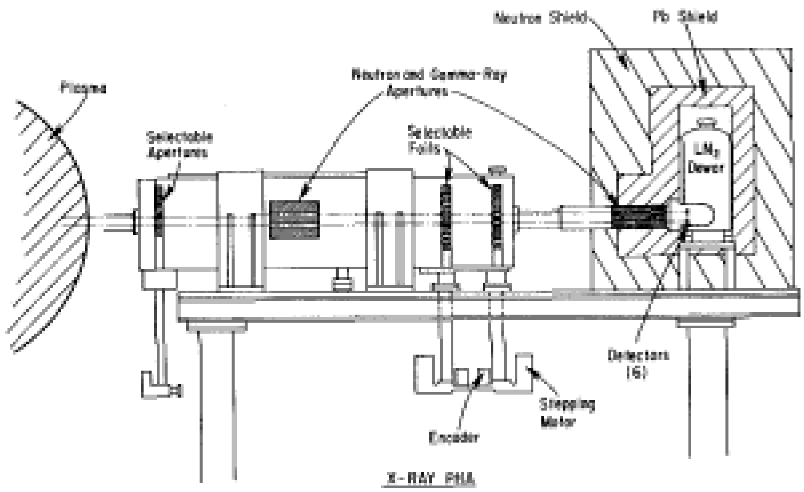


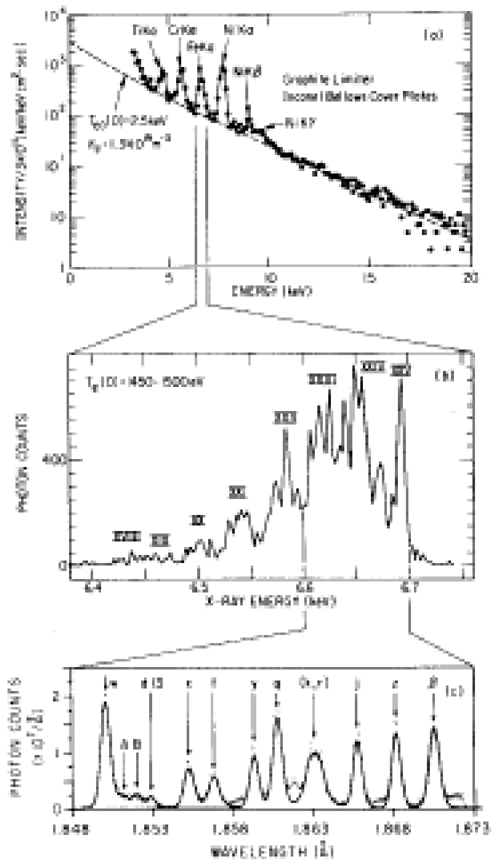
Typical spectra in this region'

Below 2.5 keV one obtains a good straight line fit.. Continuum mostly from free - free and free - bound, from impurities (e.g. oxygen). This is because of Z^2 dependence in the bremsstrahlung formula, and the exponential factor in the recombination formulae. Need careful, many-frequency measurements to avoid errors.

X-ray pulse height analysis.

The X-ray pulse height analysis provides an overview of the X-ray spectrum. Each x-ray photon generates a voltage pulse whose amplitude is proportional to the photon energy. The pulses are sorted and binned according to the energy by a pulse height analyzer, resulting in a spectrum in which the number of photons is plotted versus x-ray energy. In the soft x-ray region either lithium drifted silicon (Si(Li) or high purity germanium detectors are used. At higher energies scintillation detectors, usually NaI, couple to photo multipliers are more efficient.





X-ray photons falling onto 3 or more detectors are pre filtered by remotely selectable x-ray absorber foils and pairs of fixed and selectable apertures. The foils select the energy and the aperture the count rate. Multiple detectors are used to satisfy the simultaneous constraints of good energy resolution, time resolution, accuracy and capability to measure a range of time dependent phenomena. The photon intensities different by 3 orders of magnitude or more can be collected. At high count rates pulse pileup can distort the spectrum. This is when two simultaneously arriving photons are counted as one with an energy equal to the sum of the two individual ones.

Zeff measurements.

Practical case: many ion species present with charge Z_i. Continuum radiation then sum of contributions . When recombination is negligible, then (assuming equal Gaunt factors for all species) the emission is proportional to

$$\sum_i n_e n_i Z_i^2 = n_e^2 Z_{eff}$$

Z_{eff} is the factor by which the bremsstrahlung exceeds that of a simple H plasma. Use $n_e = \Sigma_i n_i Z_i$ to get

$$Z_{eff} = \frac{\sum_i n_i Z_i^2}{\sum_i n_i Z_i}$$

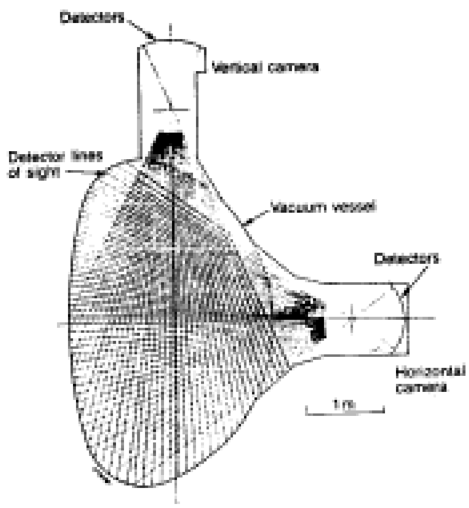
i.e. a mean ion charge. Sufficient condition for no recombination is $\hbar\omega << Z^2 R_y$, i.e. 13.6 eV for singly charged species. i.e. visible and longer wavelength. Use absolutely calibrated spectrometer. Also need independent measurement of n_e and T_e (because bremsstrahlung formula contains n_e^2 and $T_e^{-1/2}$ terms). Must ensure no lines are present in the spectra.

Note that for a thermal plasma Kirchoff's law relates absorption of radiation by inverse bremsstrahlung to emission. Because of the ω^2 dependence of the radiation (for $\hbar\omega << T$) then absorption is more important at long wavelengths. Generally waves do not propagate below the plasma frequency. For very high density plasmas as the frequency is lowered, the optical depth due to bremsstrahlung process may exceed 1 before cutoff is reached. In this case Te can be deduced fro the black body emission (as for cyclotron radiation).

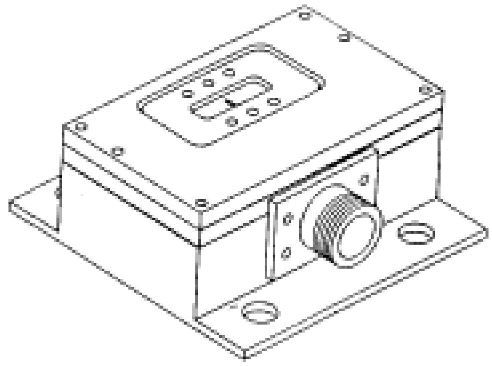
X-ray tomography

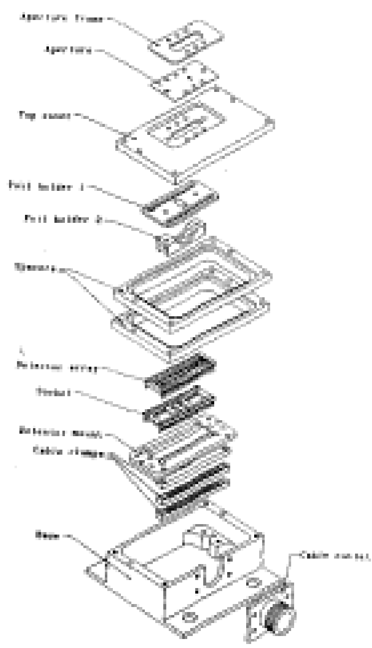
Radiation is maximal when photon energy is around electron temperature. At higher energies the exponential term dominates, and for lower energies ω and $\Delta\omega$ are smaller, so that although power per unit frequency increases, the total does not. The radiation is suitable for temporally resolved measurements using solid state detectors (semiconductor diodes like surface barrier detectors in which charge is liberated and collected, proportional to the total energy incident on the diode above a certain energy). Employing a foil (e.g. Be) allows some energy selection.

Use linear PIN photo diode arrays manufactured by EG and G (e.g. a 40-pin ceramic package for $200). The chip has 38 detectors each 4 mm by 0.94 mm with a center to center spacing of 1.00 mm. The chip is 50 mm long, and the box to hold the chip is 9 cm by 5 cm high (allowing for 'stuff'). The arrays can now fit anywhere you want. The chip fits into a standard DIP socket. Typically a 50 μm Be foil is used. Signals are 0.1 to 100 μA, and are led out using vacuum compatible coax cables with shields at circuit ground. Circuit and vessel ground are separate. This requires a ceramic coating process on the stainless steel clamping components. The detectors run in the photo voltaic mode, meaning that the bias voltage is zero volts (circuit ground). This eliminates dark current problems and still gives a fast time response. The circuitry is simple, just a trans-impedance pre amplifier, followed by further amplification. All detectors can be calibrated using a commercially available x-ray tube. A fixed diode is used as a beam monitor and the arrays are mounted on a micro controlled translation stage.



Old system using large diodes





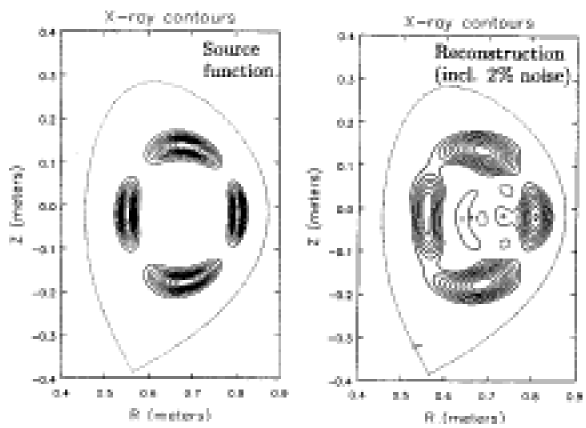
New systems are very portable.

The raw data provided is a chord integrated brightness - a more useful parameter is the localized x-ray emissivity. Since X-ray emission depends on n, T and impurities, the emissivity should match the pressure surfaces. The reconstruction or inversion problem is identical to medical scanning reconstruction. Sometimes the plasma rotates, and assist in the reconstruction. During the 1970's the computerized tomographic (CT) scanner reconstructed data to 1 mm resolution using >100,000 chords (rotating body). Although plasma physicists do not have this many chords, they also don't need 1 mm resolution.

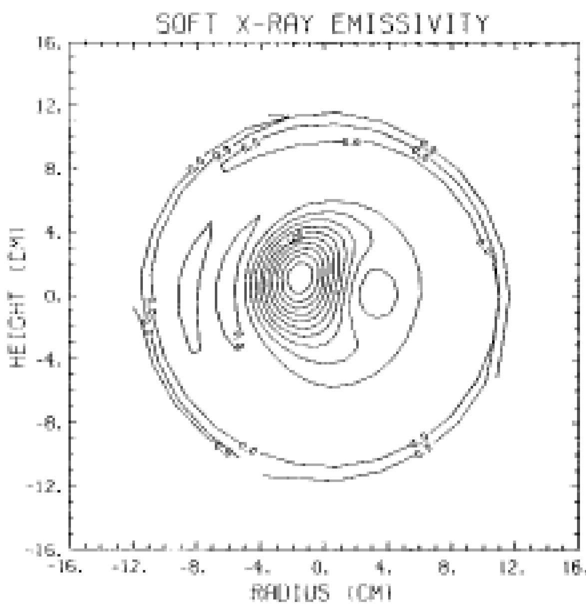
The standard algorithm is due to McCormack. If you have just one vertical, and one horizontal, array, then no matter how many chords in each array, can resolve the m =

0, sinθ, cos θ and cos 2θ, but no more. If you need to resolve up to the mth poloidal harmonic (sin and cos) then at least m+1 arrays are needed. To get a radial resolution of 0.1 a, about 40 chords per array are needed. These numbers are equivalent to a Nyquist sampling theorem.

For a five array system, each with 38 chords, poloidal modes up to 4 should be resolvable. The example shows what will happen for a given emissivity when 2% random noise is added to each channel, and the resulting emissivity is then reconstructed.



Simulated error effects



Data

Hard X-rays.

High energy electrons collide with solid surfaces and produce thick target bremsstrahlung. Electrons are slowed down by a series of collisions, until they stop. The radiation spectra is then characteristic of the average bremsstrahlung spectra for all electron energies between incident and zero energy. i.e. average the thin target over electron energies.

The bremsstrahlung photons leave the material in which they were produced, and are collected by a NaI crystal. This is efficient at stopping the photons. In the crystal a forbidden band in which there are no electrons separates the valence and conduction bands. The addition of an activator (Tl) fills in the forbidden band with the single atom energy levels of the activator. When an incident photon enters, the electrons which gain energy from the interaction with the photon are boosted from the valence band into the conduction band leaving a number of holes. The positive holes drift to the activator sites and ionize them. The electrons are free to migrate until they recombine with the ionized sites. The resulting neutral impurity atoms are in an excited state. De excitation occurs with a photon emitted.

The NAI(Tl) crystal is followed by a photo multiplier. This is a string of dynodes at increasing higher voltage. The photo cathode is a photoelectric material that when struck by a photon with E > work function can emit with certain probability an (photo) electron. The dynodes emit a few eV electron when struck with an energetic electron. The number of emitted electrons is proportional to the energy of the original electron. The PD between dynodes will accelerate the emitted electrons and a large number of electrons will be produced at each stage. The total electron charge is then converted into a voltage output pulse by integration over an R-C circuit. This signal is then amplified.

To detect the x-rays the detector must absorb the incident photons. There are three ways this can happen, photoelectric absorption, Compton scattering and pair production. These depend on material Z and photon energy.

Photoelectric absorption occurs for < 300 keV, and the photons will deposit all their energy. The incident photon is absorbed and a photoelectron emitted from one of the atomic shells of the absorber. The electron energy is $E_e = E_{photon} - E_{binding}$, and the probability increases as a high power of Z. At higher energies Compton scattering occurs: the incident photon collides with an electron, losing energy to the electron and scattering to a different direction. Pair production occurs above 3 MeV. A positron - electron pair is created in the field of the nucleus. The creation of the anti matter positron is viewed as ejecting an electron from a negative energy state into a positive energy state, leaving a hole in the region normally filled with negative energy states. This hole is the positron. There is a gap of $2m_ec^2$ between the two energy regions, so at least 1.02 MeV is required. The photons resulting from the annihilation of the positron can escape or be re absorbed in the crystal.